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ABSTRACT

Computational models have been of interest in biology for many years and have represented a particular approach to trying to understand biological processes and phenomena from a systems point of view. Much of the early work was rather abstract and high level and probably seemed to many to be of more philosophical than practical value. There have, however, been some advances in the development of more realistic models and the current state of computer science research provides us with new opportunities both through the emergence of models that can model seriously complex systems but also the support that modern software can give to the modelling process. This paper describes a few of the early simple models and then goes on to look at some new ideas in the area with a particular application drawn from the world of mycology. Some general principles relating to how new and emerging computational techniques can help to represent and understand extremely complex models concludes the paper.

Keywords

Computational model, state machine, X-machine, agent, hybrid model, fungal infection.

1. INTRODUCTION: COMPUTATIONAL MODELS.

Computational models are models of systems inspired by the model of an information processing system, in its most common manifestation a digital computer but it is not as restrictive as that in practice.

They key issues are:

- Systems interacting with their environment,
- Information processing models,
- Models exhibiting communication and concurrency
- Models that might be amenable to automated analysis as well as simulation.

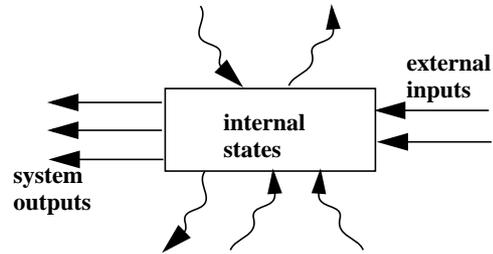


Fig. 1. A simple system interacting with its environment

The most basic discrete model: finite state machine.

- a set of internal states
- a set of external inputs - events
- a set of system outputs - actions (this is an optional feature, in some machines there are no explicit outputs)
- a transition structure to link it all together

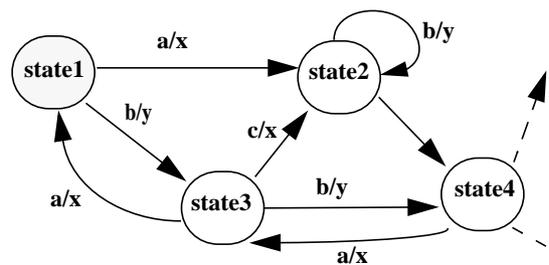


Fig. 2. A simple state machine

inputs are: **a, b, c**

outputs are: **x, y**

How does it work?

If in state1 and input (event) **a** occurs then it changes to state2 and outputs **x**;

if in state1 and input **b** occurs then it changes to state3 and output **y** occurs.

The system then waits for the next input to occur before the next state change.

This continues as long as the system continues to function.

These models have been used to analyse metabolic pathway models - Krohn, Rhodes & Langer, [5] and we can illustrate this for the simple Krebs (tricarboxylic acid) cycle.

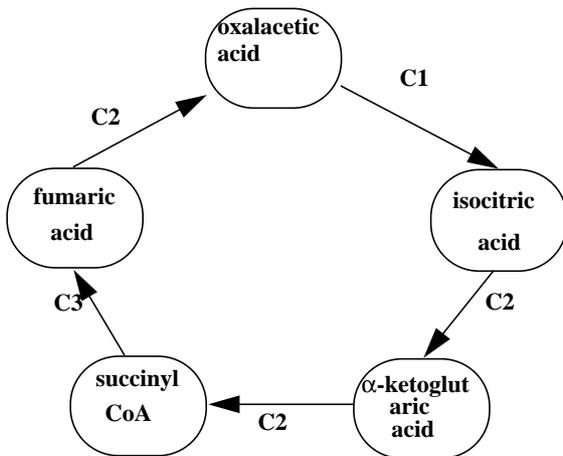


Fig. 3 A model of the Krebs cycle.

If we regard the inputs as being C1, C2 and C3 which are specific coenzymes that drive the cycle and the intermediate substrates as being the states of the system then it behaves like a state machine.

Enzymes that are required for each reaction are assumed to be present in sufficient concentration.

This is a simple model of organisation which ignores many factors, such as
 reaction kinetics,
 enzyme production,
 etc.

Several quite complex metabolic pathways have been modelled in this way.

The algebraic theory of these machines can be used to construct decompositions into simple components (generated by finite simple groups - a theory that is now well understood in mathematics). What the theory is saying is that any system of this type can be broken up naturally into a collection of sub-

systems, manufactured from simple mathematical objects, which are joined together as systems in two main ways - parallel and serial connections. For a full description of the decomposition theory and its application to the Krebs cycle see Holcombe, [2] which is an extension of the holonomy decomposition theory of Eilenberg, [6]. In the case of the Krebs cycle the algebraic structure of the machine can be decomposed to form an equivalent machine which is built up from cyclic groups of order 2 and 3 connected together as wreath products which are also combined with some simple *aperiodic* semigroups of order 2 and 3.

What does this decomposition mean, biochemically? It does not seem immediately clear and for this reason research in this area seems to have faded away. However, the impact of a greater understanding of the genomic basis for the production of the enzymes to drive the system might throw new light on the problem.

Many systems can be modelled in this way but:

- it is unsuitable for complex systems because of state space explosion;
- the functions represented by these machines are too simple for many situations;
- continuous behaviour is not modelled;
- concurrent systems are not modelled well without large problems;
- communication is hard to model this way.

Cellular automata, however, are built from these machines and have proved useful in some cases, for example:

- models of simple development;
- models of simple ecologies.

X-machines

A more sophisticated and powerful model can be introduced if we simply add an internal memory and adjust the operation of the machine to match.

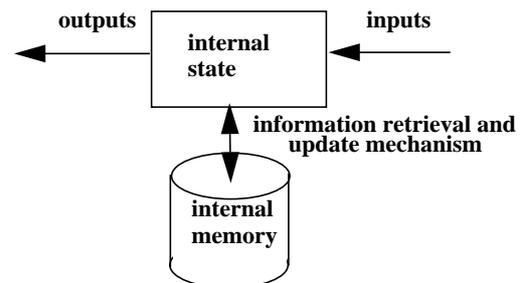


Fig. 4 A conceptual model of a stream X-machine

The system is in some state, an input \mathbf{a} is received, the initial contents of the memory are \mathbf{m} and, depending on both \mathbf{a} and \mathbf{m} , the system changes state and produces an output \mathbf{x} and updates the memory to \mathbf{m}' .

This model is the *stream X-machine* [7] and is very general.

It can model almost all computations and has been much studied in the last 5 years. The simple device of allowing an arbitrary internal memory has opened up its capabilities in a remarkable way. It is possible, using this, to model very complex systems that would, otherwise, require state spaces with millions of states. The stream X-machine model allows us to abstract these complexities and to wrap them up in hierarchical structures that can be analysed much more easily.

To obtain full Turing computability it is necessary to adapt it slightly. To model systems that operate concurrently and which communicate with each other in a more efficient way we introduce a new approach.

Communicating X-machines ([8]).

Consider a number of separate stream X-machines which have the following properties:

1. There are certain communication channels between some of the machines;
2. Some of the states in these machines are solely used for communicating messages to other machines.
3. The other states are ordinary processing states.

Each machine works separately and concurrently in an asynchronous manner until it reaches a communication state.

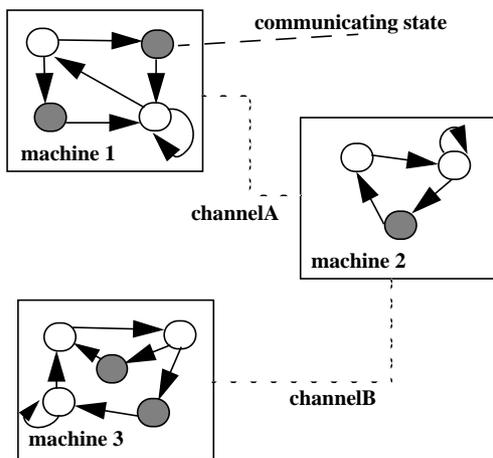


Fig. 5. A communicating X-machine system.

It then waits to either:

send a message to another machine

or receive a message from another other machine which may not be ready to send it. The receiving machine must then wait for the message.

Once a machine has been involved in a communication event successfully it can then proceed with further internal processing until it reaches the next communication state. There are a number of slight variants on this model, all, however, try to model a type of asynchronous communication in a reasonably simple and intuitive way.

The message passing between individual machines is controlled by a matrix which is illustrated in the following diagram:

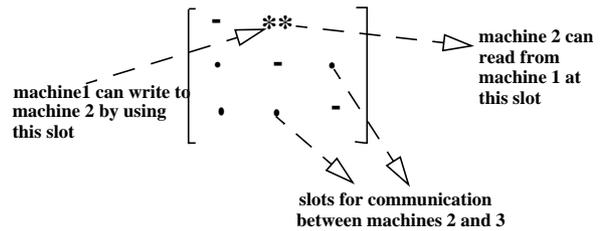


Fig. 6. The communication matrix.

2. HYBRID MACHINES.

All of the machines discussed previously are discrete, thus only instantaneous processing can be modelled and only finite discrete data is processed.

Continuous functions and real valued data cannot be incorporated into traditional finite state machine models.

The hybrid X-machine [1], overcomes this.

A hybrid machine has states and transitions as usual and responds to discrete events and performs discrete actions which are observable. The internal memory consists of:

- a set of discrete variable and
- a set of continuous variables.

When it's in a given state there are sets of equations that apply to the system's continuous variables and all the while it is in that state, with time progressing, these variables change according to these equations.

When either an appropriate external event occurs or a leaving condition is met (eg. a set point) the system moves to its next state where a different set of equations take over.

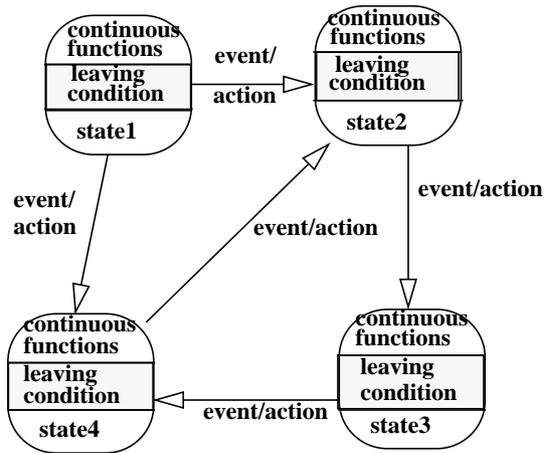


Fig. 7. A hybrid machine

Some simple examples that can be modelled this way include:

Ion flow through voltage gated channels;
Antigen-antibody interaction, [1].

The continuous variables can exhibit complex behaviour.

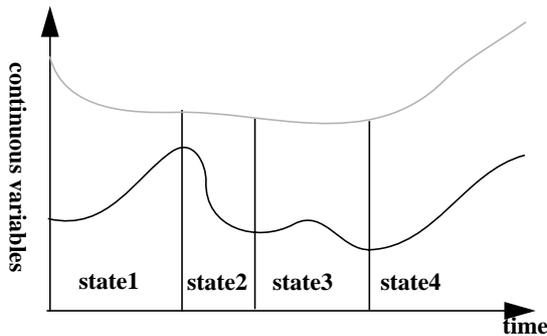


Fig. 8. The behaviour of a variable of a hybrid machine.

The equations are often composed of relatively simple functions compared to the equations that try to describe the complete functions over all states.

3. AGENTS

Agents are autonomous systems that interact with their environment - and other agents, using a set of rules to describe their behaviour.

They can be simple reactive systems or they can be very sophisticated with complex strategies and learning capabilities. The important aspect to them is that they are low level phenomenon whose behaviour in their environment emerges according to the rules. In a different environment the behaviour may change. Agents can co-operate or compete with one another.

Many biological processes seem to behave like agents, an interesting example is the system of ants and their behaviour as they form trails between the nest and a food source.

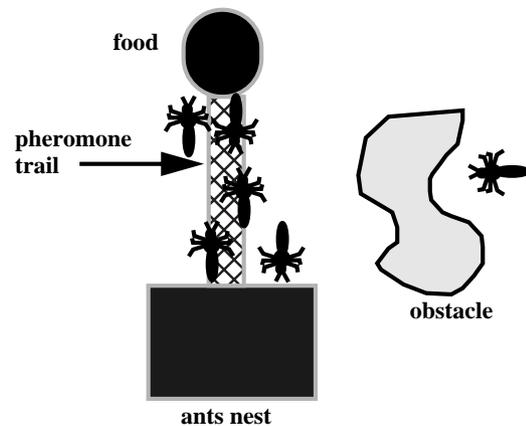


Fig. 9 A diagram of an insect trail

Suppose that these ants are simple agents that obey rules relating to the concentration of pheromone and communication events with other ants.

Rules in order of priority might be.

1. if detect obstacle then change direction
2. if detect pheromone then follow trail
3. if meet ant with food then continue
4. if find food pick up, turn round and lay trail
5. if meet ant with no food then turn round
6. if true then move randomly.

etc.

This agent can be described in full detail using an X-machine model.

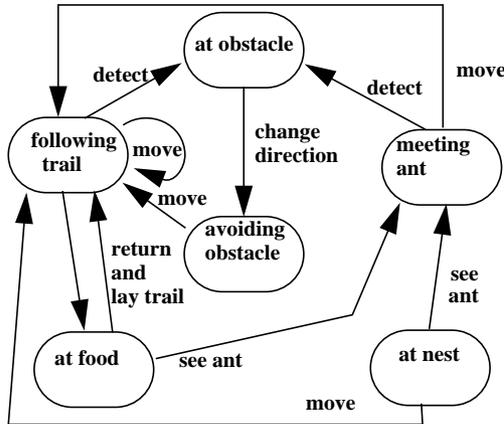


Fig. 10. An agent as an X-machine

where each function is defined by the rules and the associated inputs and memory. Memory will be the position; whether with or without food; the direction the agent is moving.

This will be a hybrid machine. Whilst in the following trail state the motion could be determined by standard equations of motion. A refinement is to introduce a probabilistic dimension to the possible state changes.

Other examples of agents can be found in the immune system, eg. T cells, B cells etc., different molecular species could also be interpreted in this way.

The next issue is how to model *Communities* of agents

Suppose that we had a collection of agents, in this case represented as individual hybrid X-machines. We now have to try to identify the communication channels and how these might work. Suppose that there are N agents and each is potentially able to communicate with any other. We thus have an $N \times N$ communication matrix.

Suppose that an agent can only communicate with other agents within a specific distance of it. There is a global memory that maintains the current position of each agent (an N -vector of coordinates). At any communication state an agent can interrogate this memory to ascertain which other agents are within communication distance. A number of strategies can be used to determine which and how many attempts at a communication can be made. The agent then puts data into the appropriate slots of the communication matrix and continues processing, moving or whatever.

Some of this data may time out if the agent it is intended for fails to retrieve it. Similarly, when an agent is within reach of another and wishes to do so it can retrieve data from the spe-

cific slot. Agents can die in which case both the row and column in the matrix become empty (void). Agents can be created, in which case the matrix is expanded to an extra row and column and the memory extended as appropriate.

Some simple ground rules must cover situations where two agents try to occupy the same co-ordinates. We should be able to simulate communities of simple agents in this way - when N becomes very large this may be a problem. Hopefully emergent properties can be identified, analytically or numerically. This needs further investigation.

4. CASE STUDY OF A HYBRID MACHINE

- *Magnaporthe griseum* (Rice blast fungus).

Rice is the world's most important food crop. This fungus destroys 40% of crops. The way that this fungus infests rice plants is a major area of research which has made significant advances in understanding the genetic basis of this process. This is a partial model of the infection stage.

It is a hybrid machine and uses some of the most recent information about the genetic basis of the behaviour of the fungus.

The spore or *conidium* is a 3 celled structure which is present in the atmosphere in affected areas. These alight on the surface of rice plants and attach themselves to the surface, this is possible despite the fact that the leaf surface is highly hydrophobic and is achieved by the conidium releasing from its tip a powerful adhesive stimulated by wetting.

The spore then germinates and produces a germ tube from one of the terminal cells of the conidium which then hooks itself into the leaf.

An *appressorium* of a roughly hemispherical shape then develops at this point of contact.

The penetration of the rice leaf surface is carried out by the build up of pressure within the appressorium until a penetration peg - part of the appressorium where it adjoins the leaf surface, is forced under this enormous pressure to penetrate the leaf surface. The pressures generated are as high as 8 MPa (or 40 times the pressure in a car tyre).

Once penetration has been achieved the fungus then forms tentacles (hyphae) within the host which then proceed to cause massive damage to the plant through the production of toxic substances and lead to the manufacture of further conidia which become the mechanism for propagation of the fungus to other plants.

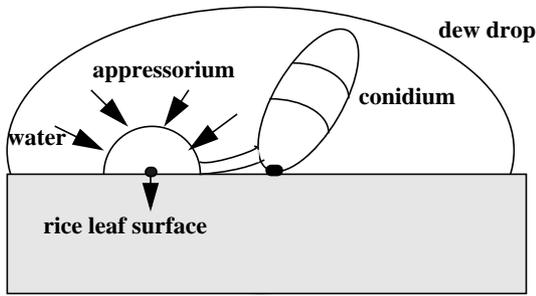
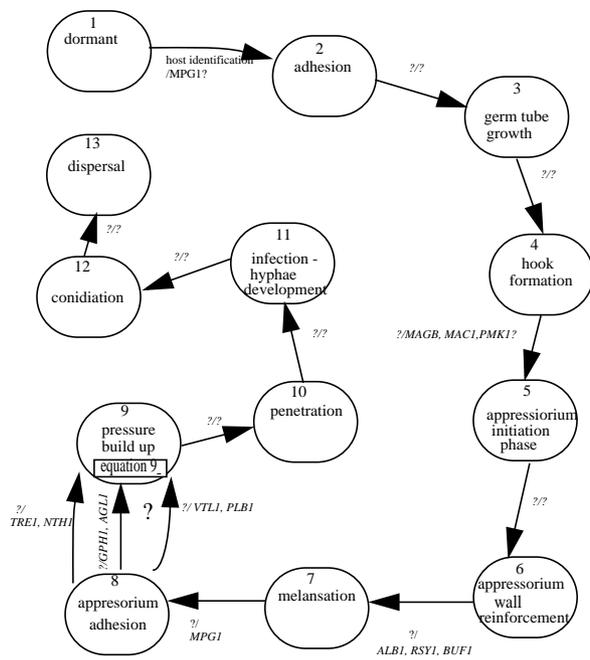


Fig. 12. Magnaporthe infection process.

Fig. 12. After N. J. Talbot, Trends in Microbiology, 9, 1995.



Legend: event/gene activity

Fig. 13. The Magnaporthe hybrid machine

The internal variables (or memory) provide information about the status of various internal aspects of the fungus, for example the concentration of glycogen is modelled and the internal pressure of the appressorium are key requirements of the model.

Each state has either a leaving condition, some condition that has to be satisfied by some internal parameter in order for a state change to occur, or there is some external event that triggers the state change.

For example, the event that occurs when a dormant spore alights on a receptive surface will trigger the initial state tran-

sition which then causes initial adherence to the receptor surface.

This is triggered by way of some dormancy-release signalling mechanism, thus activating genes needed to initiate the first process of spore tip mucilage release and adhesion.

A state transition involving a leaving condition might be state 9 → state 10 where the leaving condition is the internal appressorium pressure reaching a set point sufficient to cause the puncturing of the host leaf surface.

Whilst in state 9 this pressure is undergoing change mediated by some suitable set of equations determined by the inflow of water through the appressorium surface.

If this surface is regarded as a hemisphere and the porosity of the surface is constant throughout then the rate of increase in mass,

the pressure is given by the linear equation:

$$p(t) = \rho t/r + \text{initial pressure.}$$

During state 9, then, the pressure is increasing linearly, assuming that there is sufficient water surrounding the appressorium, until the set point is reached at which time the transition to state 10 occurs, thus triggering further gene activation for the next phase of development.

There are a number of gaps in this model which will be filled as further research into the genetic and molecular basis of the disease is carried out. It is essentially a high level model which needs to be developed in a hierarchical way so that the individual transitions actually involve complex hybrid sub-machines and these, themselves, will also break down into lower level structures until we reach an appropriate representational level.

5. CONCLUSIONS AND FURTHER WORK.

One fundamental problem in modelling such complex systems as biological systems will be trying to understand the complex interactions between many subsystems and the vastly complicated molecular and genetic activity that exists. We might be able to build these models but will we be able to understand and analyse them? It is likely that we will only be able to do this if we simplify them greatly. As an alternative approach we developed the Hybrid Projection Temporal Logic (HPTL) [1] specifically for hybrid machines. This logic allows us to define such a machine in a precise formal logic which is the first step towards using automated reasoning techniques. The basic process involves trying to establish properties about the model, now represented as a logical formula in HPTL. There are two, related, ways of doing this. First, we could try to prove theorems about the system by using theorem proving engines, this is probably impractical

since the success of automated theorem provers in dealing with extremely complex systems is limited. An alternative approach is the use of model checking techniques, [9] either alone or in combination with theorem proving. This is potentially feasible and would allow us to ask “what if” questions and query whether the system could ever get into a state with a given property holding etc. This is more feasible since model checking technology can handle models with very large state spaces. However, the technology needs to be substantially extended to cope with hybrid machines of this type. It does, however, offer a potentially rewarding direction for research.

We could, in the mean time, use simulation, *in virtuo*, to run these models and derive some useful information about the system from it.

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